**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: we have total 1 hour that is= 60 mins-10(10 mins already done) =50 mins left

Average time we have is 45 mins

For normal distribution

Z= (50-45)/8=0.625

Now probability of task will not be completed within time=1

Probability= 1-stats.norm.cdf (50,45,8) = 0.265=26.5%

🡺 B) 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS: Probability for employees age is more than 44

=1-stats.norm.cdf (44,38,6) =15.86%

Probability of employees age is between 38-44

=stats.norm.cdf (44,38,6)- stats.norm.cdf (38,38,6) =34.13%

🡺 false- p(age>44)>p(age38-44)

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS: True- probability of employee under age 30=stats.norm.cdf (30,38,6)=9.12%

🡺 9.12% of 400

🡺 9.12/100\*400 = 36.48(approx. 36)

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters

ANS: When we add two variance value gets added= *X*1 + *X*2=(2 μ,2 σ2)

But when we multiply two variance value gets squared=2 *X*1 = (2 μ,4 σ2)

Difference = (2 *X*1 - *X*1 + *X*2 )

= (2 μ,2 σ2)- (2 μ,4 σ2)

= 2 σ2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:- mean=100, alpha=0.99, std=20

Confidence Interval\_99% =stats. norm. interval (0.99,100,20) =48.483,151.516

🡺 D) (48.483, 151.516)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS: when profit added mean and variance added also, Mean=12, variance=3+4

STD=square root of ((3\*\*2)+(4\*\*2))

Doller to rupee- mean=12\*45, variance=5\*45

Probability for 95% profit range= stats. norm. Interval (0.95,540,225)

**= (99 to 980.99) in million**

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS: First 5% initial range of of profit=z score 0.05 of left tail=-1.645=540+(-1.645\*225)

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS: profit 1:- N(5, 32) 🡪 making loss mean profit lower than 0

= stats.norm.cdf (0,5,3) = 0.0477

Profit 2:- N(7, 42) 🡪 =stats.norm.cdf(0,7,4)

**= 0.0400**

Profit 1 has more probability of making loss